INVESTIGATION OF A SANDWICH TYPE CIRCULAR PLATE UNDER TRANSVERSE LOADING

A Thesis

Submitted to the Graduate Faculty

of the

University of Minnesota

By

CLARK R. GATES

In Partial Fulfillment of the Requirements

for the

Degree of Master of Science in Aeronautical Engineering

August 1948

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PREFACE

The influence of shear deformation on the behavior of a sandwich-type aircraft structural panel under load has been treated at length analytically and experimentally. However, as far as is known, experimentation is not complete in attempting to actually measure the shearing stresses causing this deformation.

A problem of interest, as pointed out by engineers of Chance Vought Aircraft, would be a study of the distribution of these shearing stresses in the core of a sandwich plate, such as Metalite. More specifically, the project the writer had in mind at the beginning of this work was to determine the distribution of transverse shear stresses along the boundaries of a simply supported rectangular Metalite panel subjected to a uniformly distributed normal load.

First, however, it was necessary to find an adequate testing method for measuring these stresses. Developing upon a unique idea suggested by Prof. J. A. Wise, University of Minnesota, such a method for experimentally determining the actual shearing stresses occurring in the core was attempted.

Time consuming difficulties arising in the perfecting of this testing procedure prevented the writer from applying it to his originally chosen problem mentioned above. However, the method was tested on a circular panel and partial success was realized, the results obtained and the testing method as developed being presented herein.

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The influence of shear deformation on the behavior of a sandwich-type aircraft structural panel under load has been treated at length analytically and experimentally. However, as far as is known, experimentation is not complete in attempting to actually measure the shearing stresses causing this deformation.

A problem of interest, as pointed out by engineers of Chance Vought Aircraft, would be a study of the distribution of these shearing stresses in the core of a sandwich plate, such as Metalite. More specifically, the project the writer had in mind at the beginning of this work was to determine the distribution of transverse shear stresses along the boundaries of a simply supported rectangular Metalite panel subjected to a uniformly distributed normal load.

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The writer wishes to acknowledge the material aid and technical assistance given him by R. A. Pitman, Boone T. Guyton, as well as others, of Chance Vought Aircraft, Dallas, Texas. Appreciation is also extended both to Prof. J. A. Wise, thesis adviser, for his suggestions and guidance in the preparation of this paper, and to E. B. Johnson for his assistance and the use of his experimental testing equipment.

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SUMMARY

This thesis presents the results of an investigation where an attempt was made to indirectly measure transverse shear stresses in the balsa core of a Metalite (sandwich-type) plate by use of a single wire electrical strain gage passed through the thickness of the core at 45°. Conditions were limited to a simply supported circular panel subjected to a uniformly distributed normal load.

Although the test method for shear stress developed herein was only partially successful, the results indicate that it may be a feasible one but further investigation is necessary to substantiate the results obtained here and to improve upon the technique used.

Consideration was also given to the deflection curve of the panel under load and to the resulting planar stresses in the faces of sandwich plate. Due to friction conditions existing between the support and the plate, the desired simple support was not fully realized. However, test deflections show fair agreement with analytical theory for the lower loading values.

INTRODUCTION

The problem under consideration in this thesis consists mainly of the development of an experimental testing method for measuring the transverse shearing stresses occurring in the balsa core at the circumferential boundary of a Metalite sandwich-type circular plate. The panel was simply supported and was subjected to a uniform normal load.

A second and subsequent issue is the measurement of these boundary stresses, by this testing method, and their comparison with analytical predictions. Also of minor concern, but still a matter of interest, are the deflection curves of the Metalite panel during the test and the stresses occurring in the aluminum faces.

Considerable analytical study, (ref. 1, 2 & 3), has been made of shear deformation, the result of shear stress in the core of sandwich material, and some experimental work has been accomplished (ref. 4 and 5). However, the distribution of shearing stresses in the core has not been completely resolved. Hence this experimental approach to the problem seemed warranted.

Once the procedure for measuring the desired stresses, as explained below under Equipment and Procedure, had been established, repeated normal loading tests in 1/4 psi. increments were made from 0 to 1-1/4 psi. on a 30 inch

diameter Metalite circular panel simply supported. Extreme care in minute adjustments were not at all times exercised where concerned with the minor issues, the effort being placed on an attempt to prove the method of test rather than obtain statistical results.

The sandwich panel used in the investigation was Metalite, produced by Chance Vought Aircraft of Dallas, Texas. Typical of most sandwich plates it has the two thin high-strength outer faces (aluminum alloy in this case) bonded to, and separated by, a relatively thick, low-density, low-stiffness core (end grain balsa). The following assumptions are made for the sandwich plate considered:

(1) Face parallel stresses in the core may be neglected so that all planar stresses are carried by the faces.

(2) The faces are very thin in comparison with the core.

(3) The neutral axis lies on the middle surface of the core.

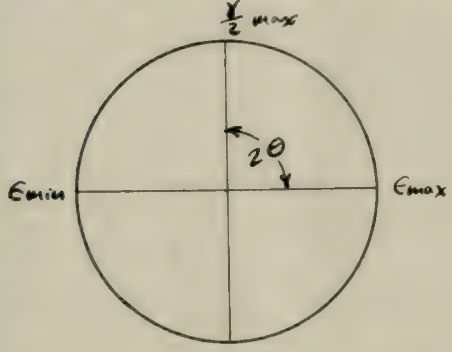
(4) Shear stresses and shear deformation in planes perpendicular to the panel may be neglected in the facings because of their relatively high shear moduli. (Later tests, after reliability of this method has been perfected, may show that the faces may carry part of the transverse shear).

(5) Transverse shear forces are carried only by the core (this, too, may be disproved later) and these shear

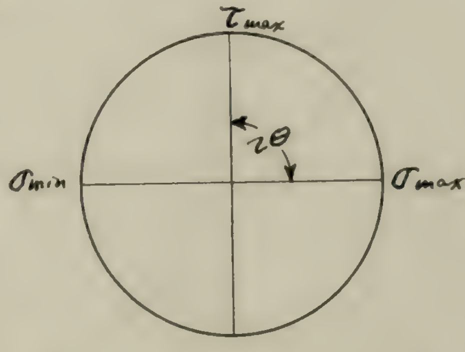
forces are distributed uniformly across the thickness of the core.

Considering a point on the neutral axis of the cross section of the sandwich material and applying the conditions prevailing there to a Mohrs circle of stress, it is seen that the shearing stress present is equal numerically to the two principal stresses and that these principal stresses are at a 45° angle to the plane of the plate. If the strain of this principal stress can be measured as it occurs in the core, then its stress can be determined and will be equal to the shearing stress at the neutral axis. Further, from assumption (5), this shearing stress will be constant across the cross section of the plate at that point.

The Mohrs circles of strain and stress mentioned above can be shown as follows:



Mohrs Circle of Strain



Mohrs Circle of Stress

From Theory of Elasticity:

$$\sigma_{max} = \frac{E}{1-\mu^2} (\epsilon_{max} + \mu \epsilon_{min}) \qquad \text{Eq. 1}$$

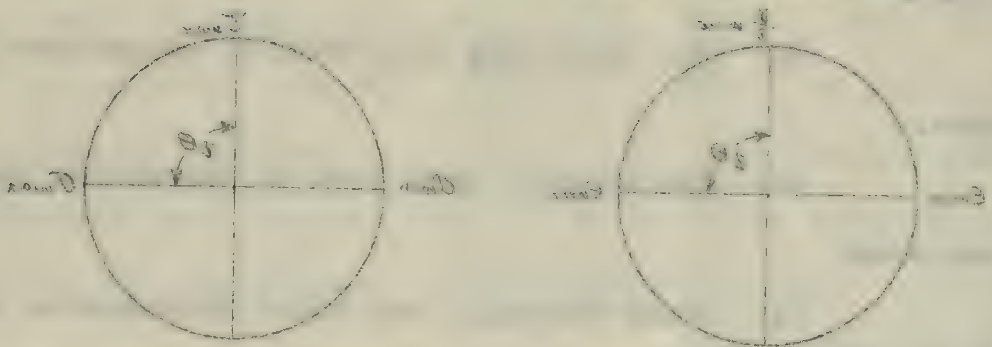
In the strain circle above,

$$\epsilon_{max} = - \epsilon_{min}$$

forces are distributed uniformly across the thickness of the core.

Considering a point on the neutral axis of the cross section of the sandwich material and applying the conditions prevailing there to a Mohrs circle of stress, it is seen that the shearing stress (stress) is equal numerically to the two principal stresses and that these principal stresses are at a 45° angle to the plane of the plate. If the strain of this principal stress can be measured as it occurs in the core, then the stress can be determined and will be equal to the shearing stress in the neutral axis. Further, from assumption (3), this shearing stress will be constant across the cross section of the plate at that point.

The Mohrs Circle of strain and stress mentioned above can be shown as follows:



Mohrs Circle of Strain   Mohrs Circle of Stress

From Theory of Elasticity:

$$\sigma_{max} = \frac{E}{1-\mu^2}\left(\epsilon_{max} + \mu\,\epsilon_{min}\right) \qquad \text{Eq. 1}$$

In the strain circle above,

$$\epsilon_{max} = -\epsilon_{min}$$

Hence, Eq. 1 changes to,

$$\sigma_{max} = -\sigma_{min} = \frac{E}{1+\mu}(\epsilon_{max}) \qquad \text{Eq. 2}$$

In regard to the shearing stresses and strains,

$$\tau_{max} = G\,\gamma_{max}$$

$$G = \frac{E}{2(1+\mu)}$$

$$\frac{\gamma_{max}}{2} = \epsilon_{max}$$

Hence,

$$\tau_{max} = \frac{E}{(1+\mu)}\,\epsilon_{max} = \sigma_{max} \qquad \text{Eq. 3}$$

Since the above development considers the material tested as homogeneous, appropriate values of E and μ must be chosen. The problem then is to install an adequate strain gage radially at a 45° angle through the thickness of the Metalite plate at the point the stress is desired to be found.

These gages were placed as near the circumferential boundary as possible so that a maximum reading would be obtained. A comparison could then be made to the loading equation for shear, i.e.:

$$\tau = \frac{p\,\pi r^2}{2\,\pi r\, t_c} \qquad \text{Eq. 4}$$

where
- p = normal load, psi.
- r = radius, in.
- t_c = core thickness, in.

The theory involving the minor issues, i.e., the panel deflection curve and the stresses in the metal faces

is taken up in Appendix A.

This investigation was carried out during the school year of 1948-1949 at the University of Minnesota, under the supervision of Prof. J. A. Wise, thesis adviser.

TESTING EQUIPMENT AND PROCEDURE

The Metalite circular panel tested was a product of Chance Vought Aircraft, Dallas, Texas. It had the following characteristics:

(1) Size - Diameter, 30 in.
Outside average thickness, 0.26 in.

(2) Core - End grain balsa. Density, 9 lb. per cu. ft. $^{+\frac{1}{2}}_{-0}$
Thickness of core, 0.25 in.

(3) Faces- 0.012 in. 75S-T6 alclad, grain of opposite faces parallel.

(4) Adhesive - Redux.

The plate had an initial domed curvature of 1/8 inch at the center.

The testing apparatus is shown in Fig. 1. A two inch thick, forty-two inch diameter circular flat steel plate was used as a base. To this base was secured a support ring made of one inch aluminum angle which had been bent on a metal shrinker to the shape of a 14-5/8 in. radius ring. The top of this angle was beaded (3/16 in. dia.) allowing minimum contact area between the support and test plate. Sealing putty was used outside the bead to assure airtightness.

By means of an aspirator, operating off from an air pressure line, air was withdrawn from the area beneath the test panel through a small hole drilled through the base plate. A second hose line from this base plate opening led to a mercury manometer, which was open to the atmosphere. By

means of a suitable scale the plate loading could be directly observed at the manometer.

Five Ames dial deflection gages were placed radially as shown in Figs. 1 and 3. Baldwin rosette strain gages were placed on the top and lower faces as shown in Fig. 3. Small holes were drilled through the vertical web of angle support to carry out the wire leads from the under side strain gages.

The special single wire strain gages (See Appendix B), set radially at a 45° angle through the core, were placed in pairs around the circumference of the test plate as shown in Fig. 2. These gages had been installed by first drilling a 45° hole through the thickness of the panel with a #70 (0.028 in. dia.) drill. A one and one-half to two inch length of the one mil gage wire (furnished by Baldwin Southwark), was threaded through the hole. Bonding glue was introduced into the hole by means of a #25G hypodermic needle and syringe, great care being exercised as the needle was moved up and down along the wire in the hole. A small amount of tension was kept on the wire during the drying process to prevent kinks and waviness. When dry, the cement acts as an insulator for the gage wire as it passes the aluminum faces. The exposed external ends of the wire were then soldered to the "K"-Box lead wires.

Use was made of the standard SR-4 Baldwin "K"-Box, the strains being read directly in micro-inches per inch. A Gage Factor of 2.00 was used for the single wire strain gages

since their Gage Factor of 1.31 could not be accommodated on the "K"-Box. (See Results and Discussion for Gage Factor correction method).

Since these single wire gages, including the dummy gage, lacked standardization in respect to length and ohms resistance, they could not all be balanced with the dummy gage on the "K"-Box. Their resistances varied from 26.2 ohms to a maximum of 57.4 ohms for the dummy gage. By placing a slide wire potentiometer in parallel with the dummy gage its resistance could be cut down to match the others and the "K"-Box could be balanced.

The testing procedure was standard for obtaining the rosette and Ames dial readings. One quarter psi. loading increments up to one and one-quarter psi were used. All gages zeroed to their original settings at the end of the tests.

The same loads were used for the single wire gages but the load was released after each reading, the zero setting checked. A loading sequence was carried through completely with one gage before moving to the next one, thereby avoiding repeated heating and cooling of the potentiometer coils. Until this potentiometer became thoroughly heated, the resulting change in resistance was very noticeable as a continual creep of the needle across the strain scale. In general, runs were not started unless the creep had dropped to less than ten micro-inches per minute with adjustments being made when

necessary for advance of the zero setting.

Due to the nature of the investigation, repeated runs were made on each single wire strain gage.

RESULTS AND DISCUSSION

1. Shear Stresses (Single Wire 45° Strain Gages)

Since the main objective of this project was to perfect and prove this method of testing, great care was taken in obtaining the strain readings (Tables I through V) of the single wire gages. Each reading was taken individually, that is, the air load was released after each reading and the zero setting of the "K"-Box checked. Repeated runs were made to determine if there was any slippage or creep in the gage itself. Gage #6A, Table 4, may have been a case where slippage occurred between the first two runs and the last four. Two of the gages, #2 and #12, were broken accidentally before repeated runs could be made.

The best average was calculated for each set of runs and was plotted with all points being shown (Fig. 4 through 9). Since the Gage Factor, 1.31, of the single wire gages, (See Appendix B), could not be accommodated on the "K"-Box, an arbitrary Gage Factor of 2.00 was used. The best average runs were then corrected to the Gage Factor of 1.31 as follows:

$$\epsilon_{(corrected)} = \epsilon_{(as\ read)} \times \frac{2.00}{1.31}$$

These final corrected values are plotted for comparison in Fig. 10. It will be noted that Gages #4 and #6A show the closest semblence of duplication. It is assumed then, for lack of better data, that their average at 1-1/4 psi, of 190 micro-inches per inch is a bonafide value. Then

[illegible]

A. Static Calibration (Single Wire [illegible] Strain Gages)

Since the main objective of this project was to perfect and prove this method of testing, great care was taken in obtaining the static readings (Series I through V) of the single wire gages. Each reading was taken individually, that is, the air load was released after each reading and the zero setting of the [illegible] checked. Repeated runs were made to determine if there was any slippage or creep in the gage itself. Gage [illegible], Table 1, may have been a case where slippage occurred between the first [illegible] and the last [illegible]. One of the gages, [illegible], was broken accidentally before repeated runs could be made.

The mean average was calculated for each set of runs and was plotted with all points being shown (Fig. [illegible] through [illegible]). Since the Gage Factor, 1.[illegible], of the single wire gages, (See Appendix [illegible]), could not be [illegible] as the "K"-Factor, an arbitrary Gage Factor of 2.00 was used. The test average runs were then corrected to the Gage Factor of 1.[illegible] as follows:

[illegible]

These final corrected values are plotted for comparison in Fig. [illegible]. It will be noted that Gages [illegible] and [illegible] show the closest condition of duplication. It is assumed then, for lack of better data, that their average at 1-1/4 [illegible], of [illegible] micro-inches per inch is a [illegible] value. Then

by use of Eq. 3 above, on page 4, the shearing stress can be determined.

The value of E for use here was selected as 250,000 psi, and μ as 0.36 (ref. 5 and 6). Inserting these and ϵ_{max} of 190 in Eq. 3 gives:

$$\tau = \frac{E}{(1+\mu)} \quad \epsilon_{max}$$

$$\tau = \frac{250,000}{(1+0.36)} \times \frac{190}{10^6}$$

$$\tau = 35 \text{ psi.}$$

For comparison, Eq. 4, page 4, gives:

$$\tau = \frac{pr}{2t} = \frac{1.25 \times 14.625}{2 \times .23}$$

$$\tau = 39.8 \text{ psi.}$$

The stress value calculated from the experimental results should be considered with caution due to the three variables involved in its computation; namely, ϵ_{max}, E, and μ. E may vary from 10,000 psi tangentially to 450,000 psi. parallel to the wood grain. Likewise, the six values of Poisson's Ratio for balsa vary from 0.009 to 0.66, depending upon the plane of the stresses under consideration. A different choice of E could have varied the result considerably. Hence before this testing method, once perfected, could be of value, a more rigorous determination of the characteristics, E and μ, must be accomplished.

Although the ϵ_{max} used above was approximately the largest obtained, there is no indication that it is a true

by use of Eq. 8 (ref), on page 4, the shearing stress can be determined.

The value of E for the bars was selected as 250,000 psi, and μ as 0.33 (ref), 0.440 (?). Inserting these and γ_{max} of 195 in Eq. 3 gives:

$$\tau = \frac{E}{(1+\mu)} \gamma_{max}$$

$$\tau = \frac{250,000}{(1+.33)} \times \frac{195}{10^6}$$

$$\tau = 36 \text{ psi.}$$

For comparison, Eq. 4, page 4, gives:

$$\tau = \frac{VQ}{Ib} = \frac{1.33 \times 11.892}{2 \times .33}$$

$$\tau = 24.5 \text{ psi.}$$

The shear value calculated from the experimental results should be considered with caution due to the three variables involved in the calculation; namely, γ_{max}, E, and μ. E may vary from 10,000 psi longitudinally to 450,000 psi perpendicular to the wood grain. Likewise, the six values of Poisson's ratio for balsa vary from 0.009 to 0.88, depending upon the plane of the strains under consideration. A different value of E would have changed the results considerably. Tensile tests, using strain gages, when performed, would be of value, a more rigorous determination of the characteristics, E and μ, must be accomplished.

Although the γ_{max} curve shows the approximately the largest strains, there is no indication that it is a true

reading. As can be seen on Fig. 10, very little duplication of readings between corresponding gages was attained. For instance, there is considerable spread among the values of the tension gages, with a still greater difference found for the compression gages. Further, it was expected that the compression-tension mates (i.e., #1 & 2, #3 & #4, etc.) would be numerically equal but of opposite sign. Gages #1 and #2 and #3A and #4 show some equality as mates but on the other hand they do not cross-check, i.e., #1 and #3, and #2 and #4.

Gage #3A's position could be strengthened if the straight line curve of #1 and #11 were extended to cross the vertical axis and then the two curves moved positively and parallel-to their present position until they were zeroed on the origin. Here again, though, #3A does not agree closely with any of the compression gages as expected.

Three factors that may have effected these results could be:

(1) The initial curvature of the plate.

(2) The fact that simple support was not completely realized, due to friction between the lower panel face and the support, thereby creating a longitudinal force in the lower face.

(3) The fact that precise loading was limited by the use of a mercury manometer.

Despite the fact that complete success of the method was not realized, it was encouraging to finally get the correct and definite indications of tension and especially compression. It is believed that this method of test

still has possibilities for further investigation. The main difficulty is concerned with obtaining a reliable bond between the core material and the complete length of the wire gage. The only glue tried was DuPont Cement #5458, ordinarily used with the Baldwin electrical strain gages. It is believed that once the glue had dried that the bond between the wire and the balsa was a permanent one (except for the possibility of #6A) since the same results were obtainable on repeated runs. However, whether the whole gage was included in this bond is not definite.

Other improvements could be made without too great difficulty by correcting the three factors mentioned two paragraphs above. Also it is suggested that the gages be moved radially inward from the support. It may be that stress concentrations from the contact area of the support were the reason for failure of the strain of the mated wires to coincide. The writer used a clearance from the support to the wire opening on the face of 1½ the thickness of the plate.

A size 75 drilled hole (0.021 in.) in the core, for threading the single wire gage, may give adequate gluing room and make a snugger fit for the wire. The #80 (.0135 in) drill originally used did not allow for ample passage of the glue while the #70 (.028 in.) drill finally used may have been too large.

One final improvement would be to make certain that the gage wire is perfectly straight when bonded. Otherwise a

subnormal tension reading will result as the kinks and slack are being taken up and absorbed with the loading.

II. Deflection and Face Stresses

The deflection data as taken with the Ames dial gages (Table VI and Fig. 13) appears reliable. However, the one-quarter pound loading increments are not precise values although they were adequate for purposes of this test. When using such low pressure values, an alcohol manometer would have been more accurate.

From the development of the deflection equation in Appendix A, it is seen that the additional deflection due to shear deformation is practically negligible in the thin panel tested, even though shear stresses were present.

Two theoretical deflection curves (effect due to shear deformation not included) are plotted on Fig. 13 with the test deflection curves. While fairly close agreement is found for the 1/4 psi curve, a large difference is noticed for the 1-1/4 psi loading curve. It is believed that this discrepancy is due almost entirely to the support friction factor mentioned on page 12. An increase in normal load would increase the effect of the longitudinal load and the plate deflection would tend to be less. The strains as obtained from the rosettes were converted to principal stresses by use of Ref. 7 (See Tables VII, VIII, and IX, and Figs. 11 and 12). The readings and results are believed to be accurate and reliable.

CONCLUSIONS

(1) This single wire gage method of test for determining shearing stresses shows positive signs of workability. However, its validity and reliability can not yet be accepted until present results are substantiated by further investigation.

(2) Tests are also needed to establish suitable core values of E and μ to be used in conjunction with this test procedure.

(3) The results obtained herein were materially effected both by the original inherent curvature of the plate and the fact that simply supported conditions were not realized.

APPENDIX A

THEORY OF FACE STRESSES & DEFLECTION

I. Face Stresses

From the theory of elasticity, the equations for stresses in the cross-section of a rectangular plate are:

$$\sigma_x = \frac{Ez}{1-\mu^2}\left[\frac{\partial^2 \zeta}{\partial x^2} + \mu \frac{\partial^2 \zeta}{\partial y^2}\right] \qquad \text{Eq. A1}$$

$$\sigma_y = \frac{Ez}{1-\mu^2}\left[\frac{\partial^2 \zeta}{\partial y^2} + \mu \frac{\partial^2 \zeta}{\partial x^2}\right]$$

ζ = deflection.
z = vertical distance from neutral axis to stress point.

Converted to polar coordinates and a circular plate, the stress equations are:

$$\sigma_t = \frac{Ez}{1-\mu^2}\left[\frac{1}{r}\frac{\partial \zeta}{\partial r} + \mu \frac{\partial^2 \zeta}{\partial r^2}\right] \qquad \text{Eq. A2}$$

$$\sigma_r = \frac{Ez}{1-\mu^2}\left[\frac{\partial^2 \zeta}{\partial r^2} + \frac{\mu}{r}\frac{\partial \zeta}{\partial r}\right] \qquad \text{Eq. A3}$$

Applying the LaGrange equation for the deflection, ζ, and a loading, p, it can be shown (ref.8) that for the circular plate,

$$\zeta = Ar^2 + C + \frac{pr^4}{64N} \qquad \text{Eq. A4}$$

where,
r = radius
N = flexural stiffness.

Substituting in Eq. A3,-

$$\sigma_r = \frac{Ez}{1-\mu^2}\left[2A(1+\mu) + \frac{4pr^2}{64N}(3+\mu)\right] \qquad \text{Eq. A5}$$

Applying boundary conditions to Eq. A5,

when $r = R$, $\sigma_r = 0$ -

$$A = -\frac{2pR^2}{64N} \times \frac{3+\mu}{1+\mu} \qquad \text{Eq. A6}$$

Applying boundary conditions to Eq. A4,

when $r = R$, $\zeta = 0$,

$$C = \frac{pR^4}{64N} \times \frac{5+\mu}{1+\mu} \qquad \text{Eq. A7}$$

Thus for outer fibre, top face, Eq. A5 becomes,

(See Fig. 2 for symbols)

$$\sigma_r = -\frac{E_f(h_c + 2t_f)p}{2(1-\mu^2)\,64N}\left[(3+\mu)(R^2 - r^2)\right] \qquad \text{Eq. A8}$$

Let[(1)] $N = \dfrac{E_f t_f h^2}{2(1-\mu^2)}$

$$\sigma_r = -\frac{(h_c + 2t_f)p}{16 t_f h^2}\left[(3+\mu)(R^2 - r^2)\right] \qquad \text{Eq. A9}$$

Applying Eq. A9 to the plate tested, where,

$E = 10^7$ psi $\qquad \mu = \frac{1}{3}$

$h_c = 0.236$ in (0.23" balsa core plus redux bond)

$t_f = 0.012$ in. $\qquad h = h_c + t_f = 0.248$ in.

$$\sigma_r = 73.4p(r^2 - R^2) \qquad \text{Eq. 10}$$

(1) Seide, Paul and Stowell, E.Z.; Elastic and Plastic Buckling of Simply Supported Metalite Type Sandwich Plates, NACA Tech. Note 1822, February 1949.

Taking the point of rosettes #2 and #3, where the radius is 7.3125 inches and using a loading of 0.25 psi.,

$$\sigma_r = \pm 2943 \text{ psi.}$$

This shows fair agreement with test results of Tables VIII and IX, page 33 and 34, where values are,

$$\sigma_r \cong -2823 \text{ psi. compression (top face)}$$

$$\sigma_r \cong +2575 \text{ psi Tension (lower face)}$$

At 0.50 psi. load, calculated gives,

$$\sigma_r = \pm 5,885 \text{ psi}$$

while test shows,

$$\sigma_r \cong -4230 \text{ psi}$$

$$\sigma_r \cong +4840 \text{ psi}$$

At 0.75 psi., calculated shows,

$$\sigma_r = 8830 \text{ psi.}$$

while tests gave,

$$\sigma_r \cong -5770$$

$$\sigma_r \cong +7650$$

As can be seen the descrepency between the test results and the analytical results is getting larger with increased load due to the failure to achieve a simple support. The effects of the inherent longitudinal load are increasing with the load.

II. DEFLECTION

To obtain the deflection equation (no shear deformation) apply the constants A and C, Equations A6 and A7 to the deflection Equation A4, giving,

$$\delta = \frac{p}{64N}\left[r^4 - 2\,\frac{3+\mu}{1+\mu}\left(r^2R^2\right) + \frac{5+\mu}{1+\mu}R^4\right] \qquad \text{Eq. A11}$$

Applying the constant test plate values to Eq. A11, the following deflection equation is obtained,

$$\delta = 0.0000003767\,p\left[r^4 - 1070\,r^2 + 183{,}200\right] \qquad \text{Eq. A12}$$

or at the center,

$$\delta = 0.690\,p \qquad \text{Eq. A13}$$

Considering deflections due to shear as found in reference 9, page 143, but using a constant cross-section value instead of a parabolic curve, the shear deflection is,

$$\delta_{shear} = \frac{1}{4}\,\frac{p}{G\,h}\left(R^2 - r^2\right) \qquad \text{Eq. A14}$$

Using a shear modulus for Metalite of this core and face of G = 29,000 psi. from reference 5, Fig. 4.01, at the center Eq. A14 gives,

$$\delta_{shear} = 0.00674\,p$$

Total center deflection will be,

$$\delta = \left(0.690 + 0.00674\right)p$$
$$= 0.69674\,p$$

Or, in the case of the thin panel tested, the deflection due to shear is only 0.97% of the total, or just less than one percent.

Neglecting this negligible shear deflection and considering Eq. A11 alone, the following values for δ are found for the 0.25 psi., the 1.00 psi., and the 1.25 psi. loading -

Table of Deflections

Radius(in)	Loading(psi)		
	0.25	1.0	1.25
0	0.1725	0.690	0.863
3.75	0.1588	0.635	0.794
6.3125	0.1337	0.535	0.669
10.00	0.0813	0.325	0.406

APPENDIX B

Problems Concerning the Use of the Single Wire Strain Gage

Numerous difficulties were encountered in attempting to perfect the installation of the single wire strain gage for use in this testing method. The greatest of these was obtaining a complete and reliable bond between the wire and the core material.

As mentioned in the body of this report, a 0.0135 inch hole was first tried as a conduit through the core for the gage. After the gage wire was threaded through the hole, glue was placed over the opening on each face of the panel. The wire was then slowly drawn back and forth through the core so that the glue would be carried in through the length of the conduit. However, loading tests indicated that insufficient glue was reaching the interior, since all gages slipped with the first loading.

An attempt was made to correct this difficulty by heating the gage wire which in turn would heat and loosen the glue into better distribution. Heating was attempted by passing current through the wire, but it did not prove at all practical, the wire gages being too delicate. Suction, to draw the glue through the core, was then tried on the 0.0135 inch hole. In this case, the quick-drying glue would start hardening as soon as it reached the opposite face, thereby blocking the passage of sufficient glue for

bonding. A change to a larger hole, 0.028 inch, corrected this trouble, but still not enough of the glue was being distributed inside for satisfactory bonding purposes.

The most satisfactory gluing method tried and the one the final results were obtained from, was to employ a hypo needle to introduce the glue into the gage hole, as mentioned earlier in the report. Positive results were obtained in all cases, although very weak in some instances. A re-gluing of these weak ones showed marked improvement, the old bond being loosened with a hypo injected solvent (acetone). Although this last method gave partial success it can not be accepted until better duplication of results is obtained.

Another problem mentioned in the report is the desirability of having all the single wire gages of the same resistance, or within a few ohms, of the dummy gage, thus eliminating the use of the potentiometer or additional resistance.

The exposed portions of the gage wire should be shielded from drafts since sudden temperature changes make a noticeable effect on strain readings being taken from the "K"-Box.

Since the Gage Factor, needed for use with the "K"-Box, of this single gage wire was unknown, it had to be determined. The Gage Factor is a constant for each type and size of wire and is:

$$G.F. = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}}$$

Since it was impractical to measure these factors for the wire concerned, the following means was employed. A one-wire paper-covered strain gage was made up on a steel rod whose loading-strain curve was known. Following the pattern of the standard gages, this special one was made by first gluing a piece of rice paper to the rod and then stretching a length of the wire longitudinally over the paper. More glue was applied and a top cover of rice paper placed over all.

When the gage was dry, the "K"-Box leads were soldered to the exposed ends and the rod was then tested in tension in a Riehle Testing Machine. Three arbitrary Gage Factor settings were used on three loading runs (Table X).

From the plotted results (Fig. 14), the Gage Factor of the wire gage under test was determined by comparing the three test curves to the known strain curve, the relationship being a direct proportion.

Thus,

Run 1 $\quad G.F. = \frac{880}{1200} \times 1.77 = 1.30$

Run 2 $\quad G.F. = \frac{780}{1200} \times 2.04 = 1.33$

Run 3 $\quad G.F. = \frac{710}{1200} \times 2.20 = 1.30$

Average $\quad 1.31$

Hence, the G.F. for this case is 1.31.

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TABLE I

SINGLE WIRE STRAIN GAGES

GAGE READINGS (Micro inches)

Gage # 1

Load psi	Run 1	Run 2	Run 3	Run 4	Best Average	Corrected for Gage Factor
0.00	0	0	0	0	0.0	0.0
0.25	5	2	3	4	3.5	5.3
0.50	20	14	11	20	16.2	24.9
0.75	40	28	30	32	30.0	45.8
1.00	35	43	45	42	43.3	60.1
1.25	55	62	58	58	58.2	89.0

Gage # 2

Load psi	Run 1	Run 2	Run 3	Run 4	Best Average	Corrected for Gage Factor
0.00	0	Wire			0.0	0.0
0.25	-	was			-	-
0.50	-20		broken		-20.0	-30.8
0.75	-40				-40.0	-61.1
1.00	-50				-50.0	-76.4
1.25	-60				-60.0	-91.6

TABLE II

SINGLE WIRE STRAIN GAGES

GAGE READINGS (Micro inches)

Gage # 3

Load psi	Run 1	Run 2	Run 3	Run 4	Run 5	Best Average	Corrected for Gage Factor
0.00	0	0	0	0		0.0	0.0
0.25	0	2	1	2		1.7	2.6
0.50	3	6	6	6		5.2	7.9
0.75	5	8	0	7		6.7	10.2
1.00	13	11	12	14		12.5	19.1
1.25	18	19	17	20		18.5	28.2
Gage # 3_A (Reglued)							
0.00	0	0	0	0		0.0	0.0
0.25	26	20	24	25		23.8	30.4
0.50	31	28	33	30		30.5	46.0
0.75	39	40	40	39		39.5	60.4
1.00	53	60	64	57		58.5	89.4
1.25	80	80	80	84		81.0	123.7
Gage # 4							
0.00	0	0	0	0	0	0.0	0.0
0.25	-50	-27	-42	-33	-36	-37.6	-57.4
0.50	-62	-56	-52	-50	-	-55.0	-84.0
0.75	-76	-90	-90	-73	-80	-81.8	-125.0
1.00	-105	-100	-107	-105	-	-104.2	-159.0
1.25	-112	-124	-137	-118	-112	-120.6	-184.0

TABLE III

SINGLE WIRE STRAIN GAGES

GAGE READINGS (Micro inches)

Gage # 5

Load psi	Run 1	Run 2	Run 3	Best Average	Corrected for Gage Factor
0.00	0	0	0	0.0	0.0
0.25	8	9	-	8.5	13.0
0.50	15	15	-	15.0	22.9
0.75	22	21	-	21.5	32.8
1.00	32	30	-	31.0	47.4
1.25	44	39	40	41.0	62.6

Gage # 6

Load psi	Run 1	Run 2	Run 3	Best Average	Corrected for Gage Factor
0.00	0			0.0	0.0
0.25	-5			-5.0	-7.6
0.50	-8			-8.0	-12.2
0.75	-10			-10.0	-15.3
1.00	-12			-12.0	-18.3
1.25	-13	-13		-13.0	-19.9

TABLE IV

SINGLE WIRE STRAIN GAGES

GAGE READINGS (Micro inches)

Gage # 6_A (# 6 Reglued)

Load psi	Run 1	Run 2			Average 1 & 2	Corrected for Gage Factor
0.00	0	0			0.0	0.0
0.25	-38	-29			-33.0	-50.9
0.50	-58	-48			-53.0	-81.0
0.75	-112	-75			-81.0	-123.8
1.00	-113	-87			-104.0	-158.8
1.25	-140	-120			-130.0	-198.6
Gage # 6_A (Continued)	**3**	**4**	**5**	**6**	*	
0.00	0	0	0		0.0	0.0
0.25	-31	-33	-32		-32.0	-48.9
0.50	-48	-50	-49		-48.5	-74.1
0.75	-62	-61	-62		-61.7	-95.0
1.00	-78	-75	-80	-72	-76.2	-114.8
1.25	-79	-91	-89	-81	-85.0	-130.0

* Average of Runs 3, 4, 5, & 6.

TABLE V

SINGLE WIRE STRAIN GAGES

GAGE READINGS (Micro inches)

Load psi	Run 1	Run 2	Run 3	Run 4	Run 5	Best Average	Corrected for Gage Factor
			Gage # 10				
0.00	0	0	0	0	0	0.0	0.0
0.25	-35	-13	-10	-13	-11	-11.7	-17.9
0.50	-42	-18	-22	-13	-15	-23.2	-35.4
0.75	-110	-112	-120	-115	-	-115.7	-176.7
1.00	-107	-110	-119	-110	-	-113.0	-174.1
1.25	-113	-107	-115	-111	-	-111.0	-168.0
			Gage # 11				
0.00	0	0	0	0		0.0	0.0
0.25	-8	-8	-7	-		-7.7	-11.8
0.50	0	0	0	-		0.0	0.0
0.75	11	16	17	18		17.0	26.0
1.00	35	35	36	-		35.3	53.9
1.25	48	49	42	48		48.3	73.7
			Gage # 12				
0.00	0	-				0.0	0.0
0.25	-2	-				-2.0	-3.1
0.50	-4	-				-4.0	-6.1
0.75	-7	-				-7.0	-10.7
1.00	-13	-				-13.0	-19.9
1.25	-23	-23				-23.0	-35.2

TABLE VI

PLATE DEFLECTIONS (Inches)

Load psi	Center	3.75 in. radius	6.3125 in. radius	10.00 in. radius	14.625 in. radius
0.00	0	0	0	0	0
0.25	-0.175	-0.163	-0.138	-0.086	0
0.50	-0.300	-0.273	-0.235	-0.147	0
0.75	-0.410	-0.380	-0.323	-0.200	0
1.00	-0.489	-0.455	-0.387	-0.242	0
1.25	-0.558	-0.520	-0.442	-0.279	0
1.50	-0.624	-0.595	-0.508	-0.318	0

Note: Deflections were taken by means of Ames dials placed above the top face.

TABLE VII

ROSETTE STRAIN GAGES

GAGE READINGS (Micro inches)

Top Face

Rosette No.	Gage No.	Strain Readings For Each Loading Increment					
		0 psi	¼ psi	½ psi	3/4psi	1 psi	1¼ psi
	T1	0	-190	-340	-530	-730	-990
1	T2	0	-100	-160	-240	-310	-420
	T3	0	40	60	120	160	210
	T4	0	-160	-270	-360	-420	-470
3	T5	0	-240	-390	-550	[illegible]	-740
	T6	0	-260	-420	-570	-670	-770
	T7	0	-250	-420	-540	-610	-660
5	T8	0	-280	-450	-560	-620	-670
	T9	0	-280	-430	-550	-620	-660
Bottom Face							
	B1	0	200	310	340	320	280
2	B2	0	130	160	180	190	190
	B3	0	-60	-70	-50	-20	20
	B4	0	140	290	480	650	840
4	B5	0	250	420	620	740	[illegible]
	B6	0	230	380	500	620	[illegible]
	B7	0	220	450	690	870	1050
6	B8	0	250	460	700	880	1040
	B9	0	230	480	630	[illegible]	1010

TABLE VIII

ROSETTE STRAIN GAGES

PRINCIPAL STRAINS & STRESSES
ANGLE TO PRINCIPAL STRESS

Top Face

Load psi	e_{min}	e_{max}	σ_{min} psi	σ_{max} psi	Angle to σ_{min}
		#1 Rosette (45°)			
0.00	0	0	0	0	0
0.25	-192.7	42.7	-2006	-242	6.13° cw
0.50	-340.5	60.5	-3568	-596	2.35 cw
0.75	-531.8	126.8	-5330	-586	2.07 cw
1.00	-730.7	160.7	-7620	-935	1.51 cw
1.25	-930.7	210.7	-10,360	-1344	1.43 cw
		# 3 Rosette (120°)			
0.00	0	0	0	0	0
0.25	-281.1	-158.9	-3740	-2823	5.45° cw
0.50	-511.0	-209.0	-6510	-4230	3.29 cw
0.75	-703.7	-282.9	-8940	-5770	1.57 cw
1.00	-827.2	-326.2	-10,430	-6720	1.9[illegible] cw
1.25	-960.6	-359.4	-12,110	-7590	1.65 cw
		# 5 Rosette (45°)			
0.00	0	0	0	0	0
0.25	-294.1	-255.9	-4155	-3950	22.50° cw
0.50	-450.5	-399.5	-6560	-6190	32.34 cw
0.75	-580.8	-529.2	-8294	-8066	35.73 cw
1.00	-627.1	-607.9	-9270	-9150	22.50 cw
1.25	-679.0	-650.0	-9980	-9830	45.00 cw

(1) angle is measured clockwise, cw, or counterclockwise, ccw, from the right side of a tangential axis passed through center of rosette.

TABLE IX

ROSETTE STRAIN GAGES

PRINCIPAL STRAINS & STRESSES
ANGLE TO PRINCIPAL STRESS

Bottom Face

\# 2 Rosette (45°)

Load psi	ϵ_{min}	ϵ_{max}	σ_{min} psi	σ_{max} psi	Angle to σ_{max} (1)
0.00	0	0	0	0	0
0.25	-62.3	[illegible]	279	[illegible]	5.31°cw
0.50	-74.2	314.2	344	3255	[illegible] cw
0.75	-54.1	343.8	682	[illegible]	[illegible] cw
1.00	-24.6	324.6	941	[illegible]	[illegible] cw
1.25	14.6	[illegible]	[illegible]	[illegible]	[illegible] cw

\# 4 Rosette (120°)

Load psi	ϵ_{min}	ϵ_{max}	σ_{min} psi	σ_{max} psi	Angle to σ_{max} (1)
0.00	0	0	0	0	0
0.25	139.0	274.3	[illegible]	3593	4.95°cw
0.50	[illegible]	[illegible]	4340	[illegible]	8.30 cw
0.75	472.2	634.4	7650	[illegible]	[illegible] cw
1.00	637.1	742.9	9900	10690	20.45 cw
1.25	811.3	[illegible]	12380	12900	20.46 cw

\# 6 Rosette (45°)

Load psi	ϵ_{min}	ϵ_{max}	σ_{min} psi	σ_{max} psi	Angle to σ_{max} (1)
0.00	0	0	0	0	0
0.25	[illegible]	262.4	3433	[illegible]	13.29°ccw
0.50	442.2	490.8	6850	7095	9.21 cw
0.75	[illegible]	700.8	10160	[illegible]	54.22 ccw
1.00	837.6	882.4	12730	13070	58.29 ccw
1.25	[illegible]	1052.4	15270	[illegible]	[illegible] ccw

$E = 10^7$ psi $\mu = 1/3$

(1) Angle is measured clockwise, cw, or counterclockwise, ccw, from the right side of a tangential axis passed through the center of the rosette.

TABLE X

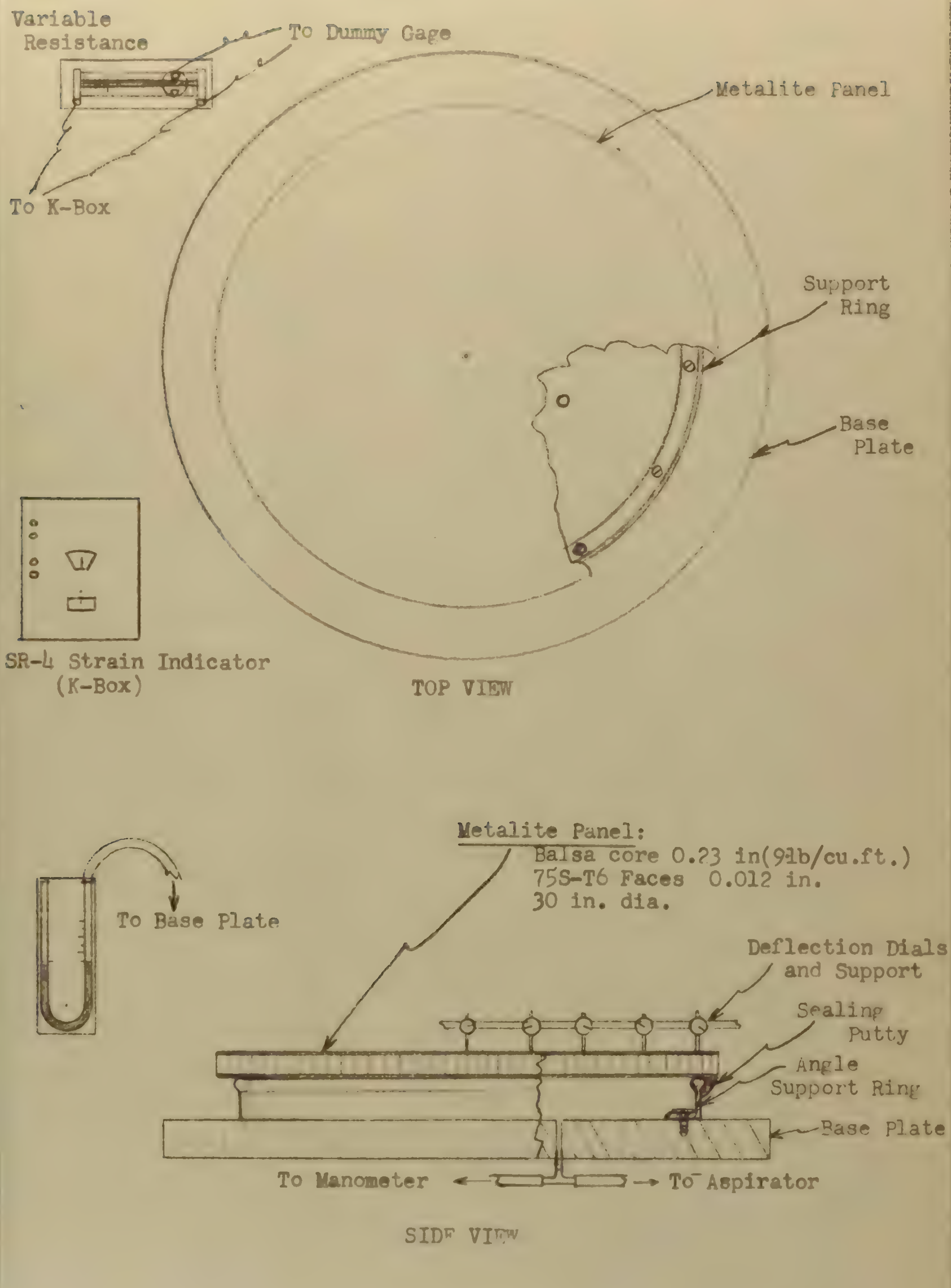
GAGE FACTOR

TENSION LOAD vs STRAIN

	Gage Factor 1.77	Gage Factor 2.04	Gage Factor 2.20
Load psi	Micro inches	Micro inches	Micro inches
0	0	0	0
2000	180	140	140
4000	350	300	280
6000	525	450	420
8000	715	610	560
10000	880	780	710

Fig. 1

TESTING EQUIPMENT



(Not to scale)

Fig. 2

SINGLE WIRE STRAIN GAGES

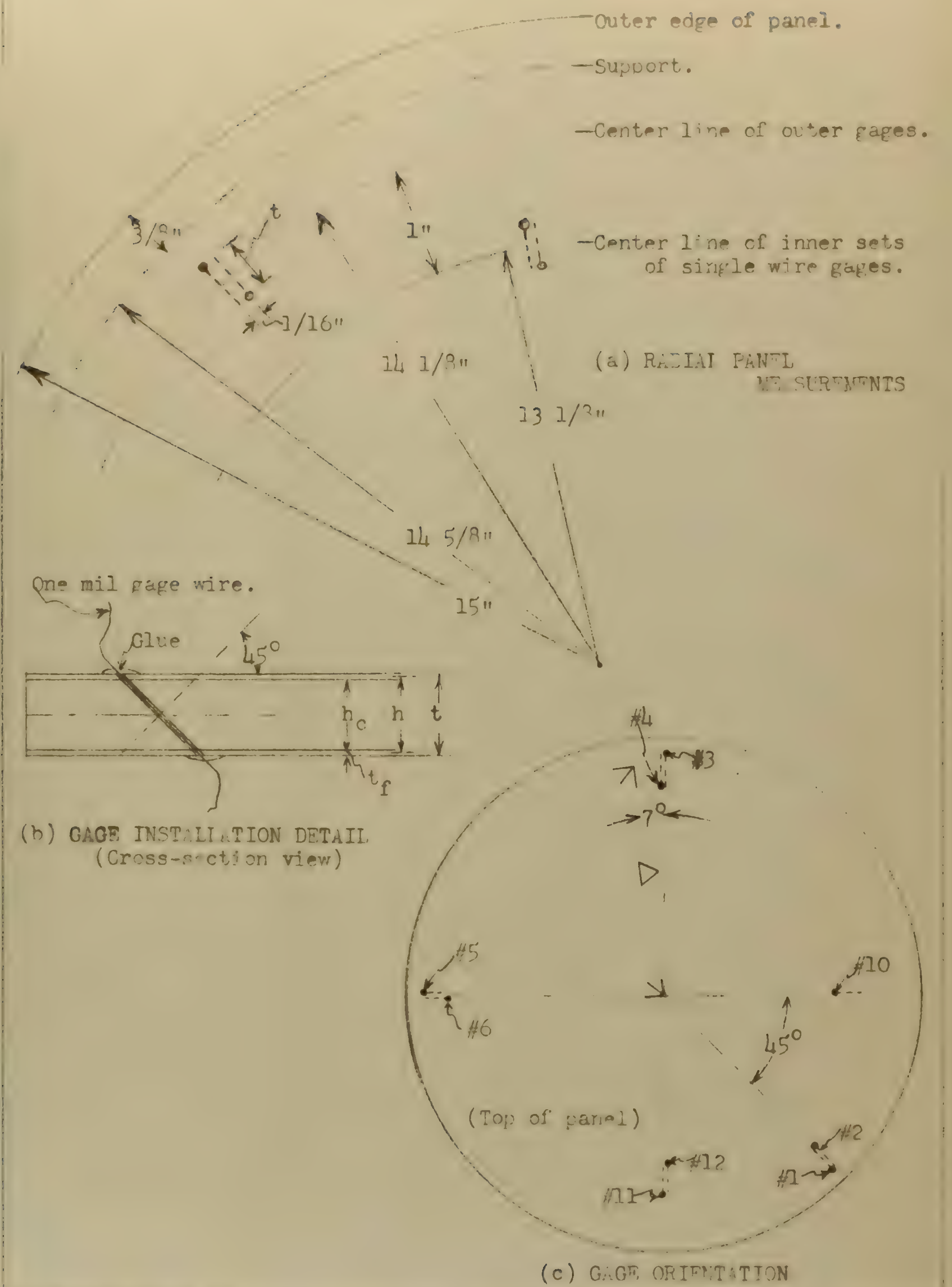
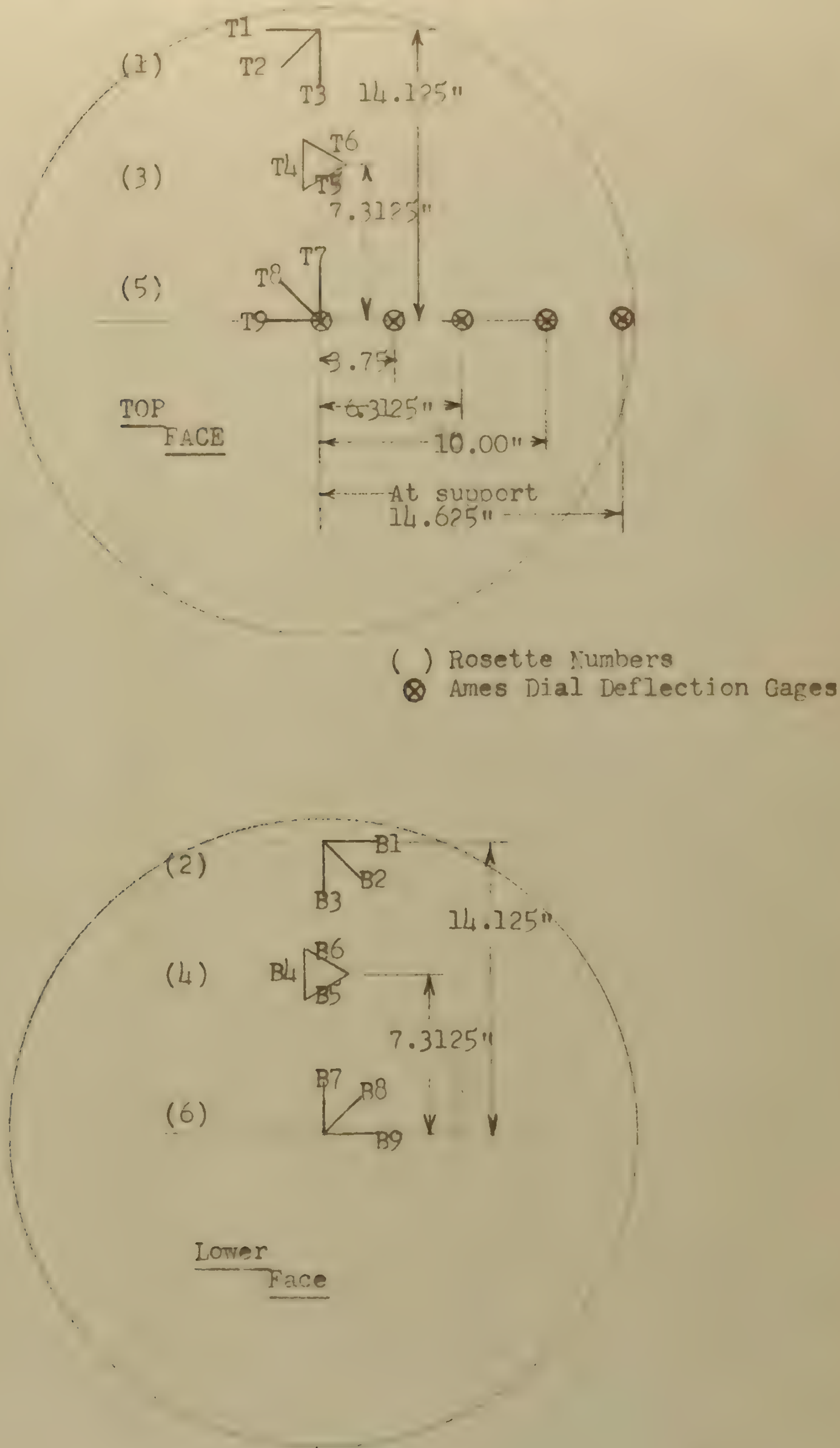


Fig. 3

POSITIONING OF ROSETTE & AMES GAGES



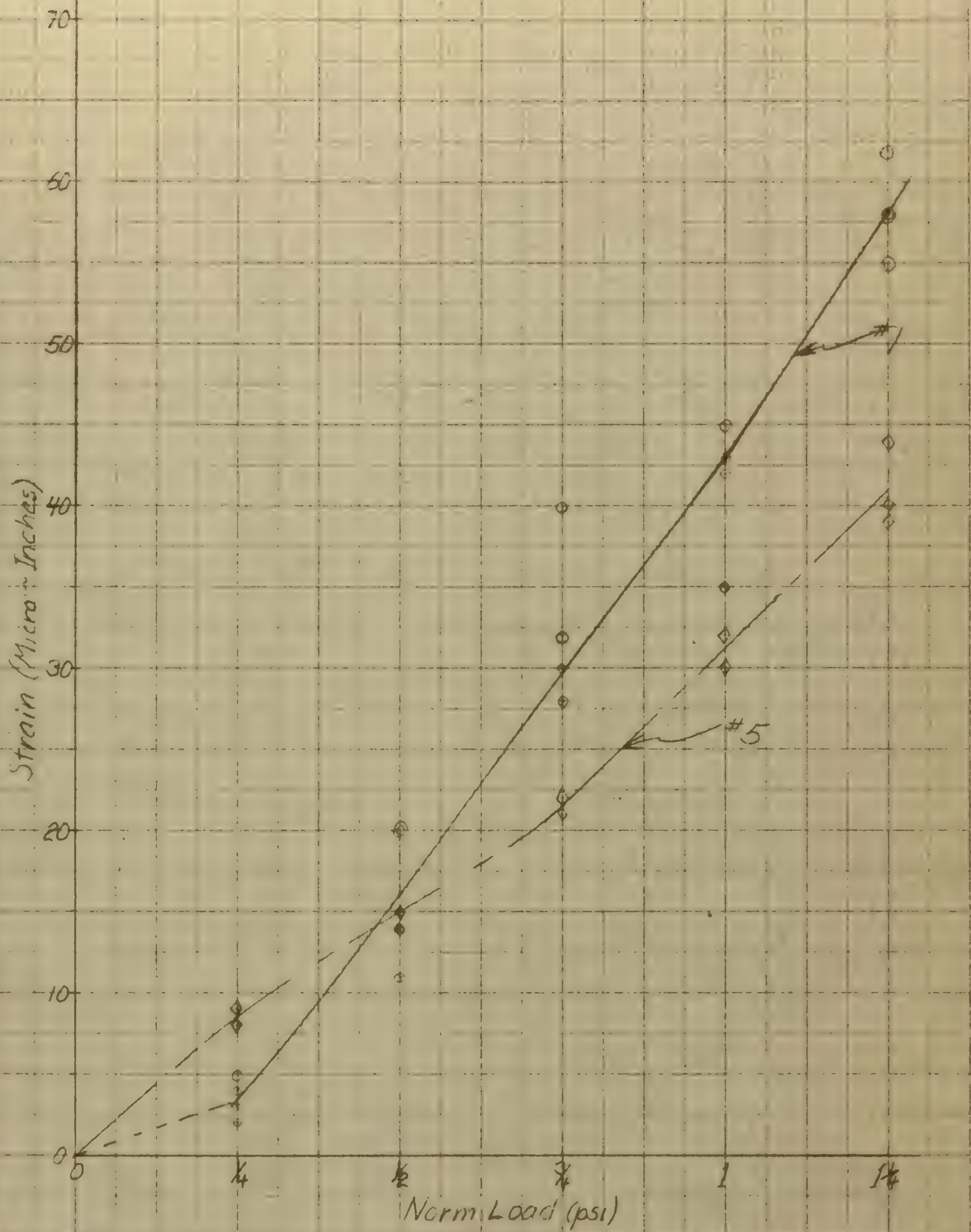


Fig. 4
Strain~Loading Curve
Gages #1 & #5
(G.F. not corrected)

CHG

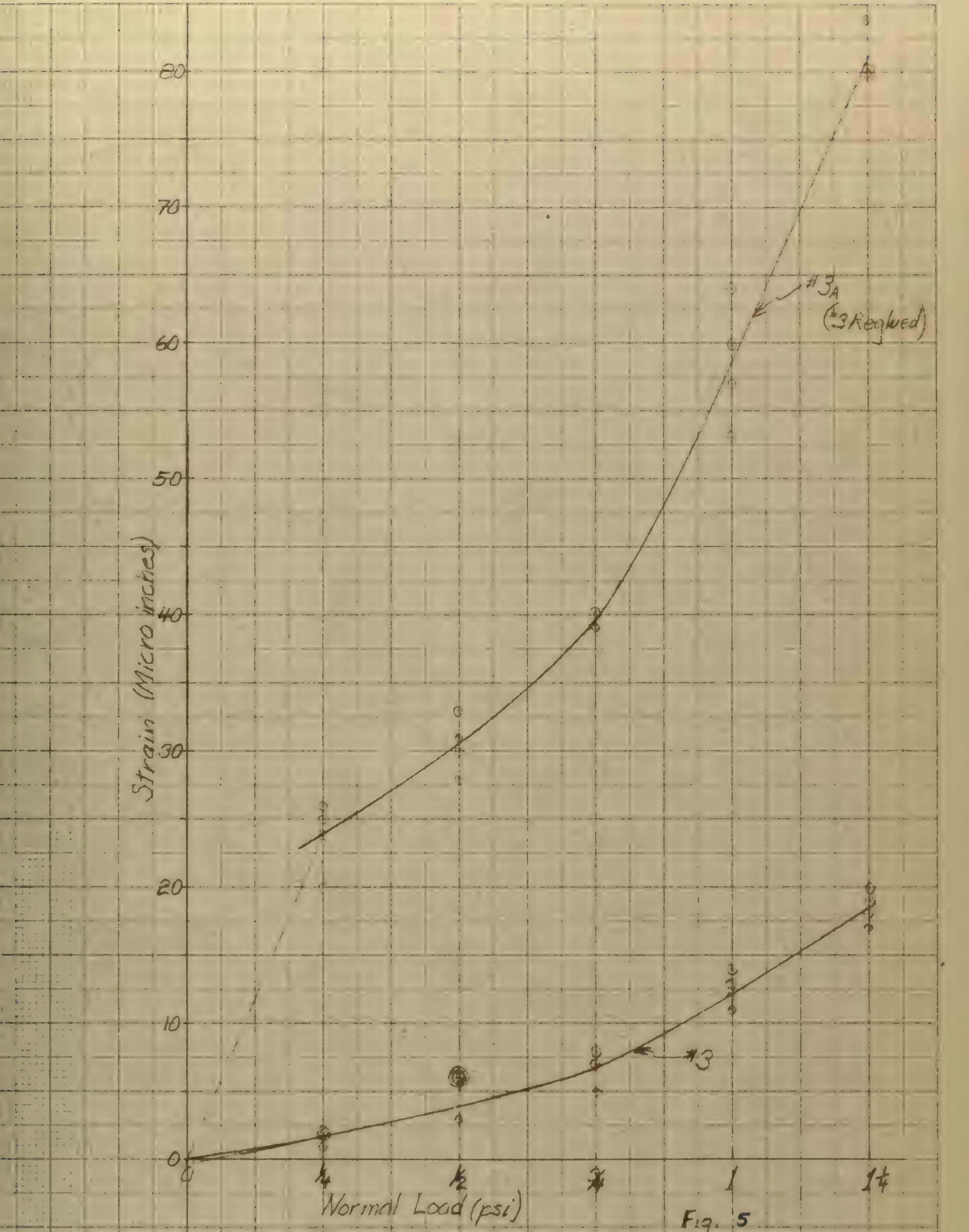


Fig. 5
Strain ~ Loading Curves
Gages #3 & #3A
(Uncorrected for G.F.)

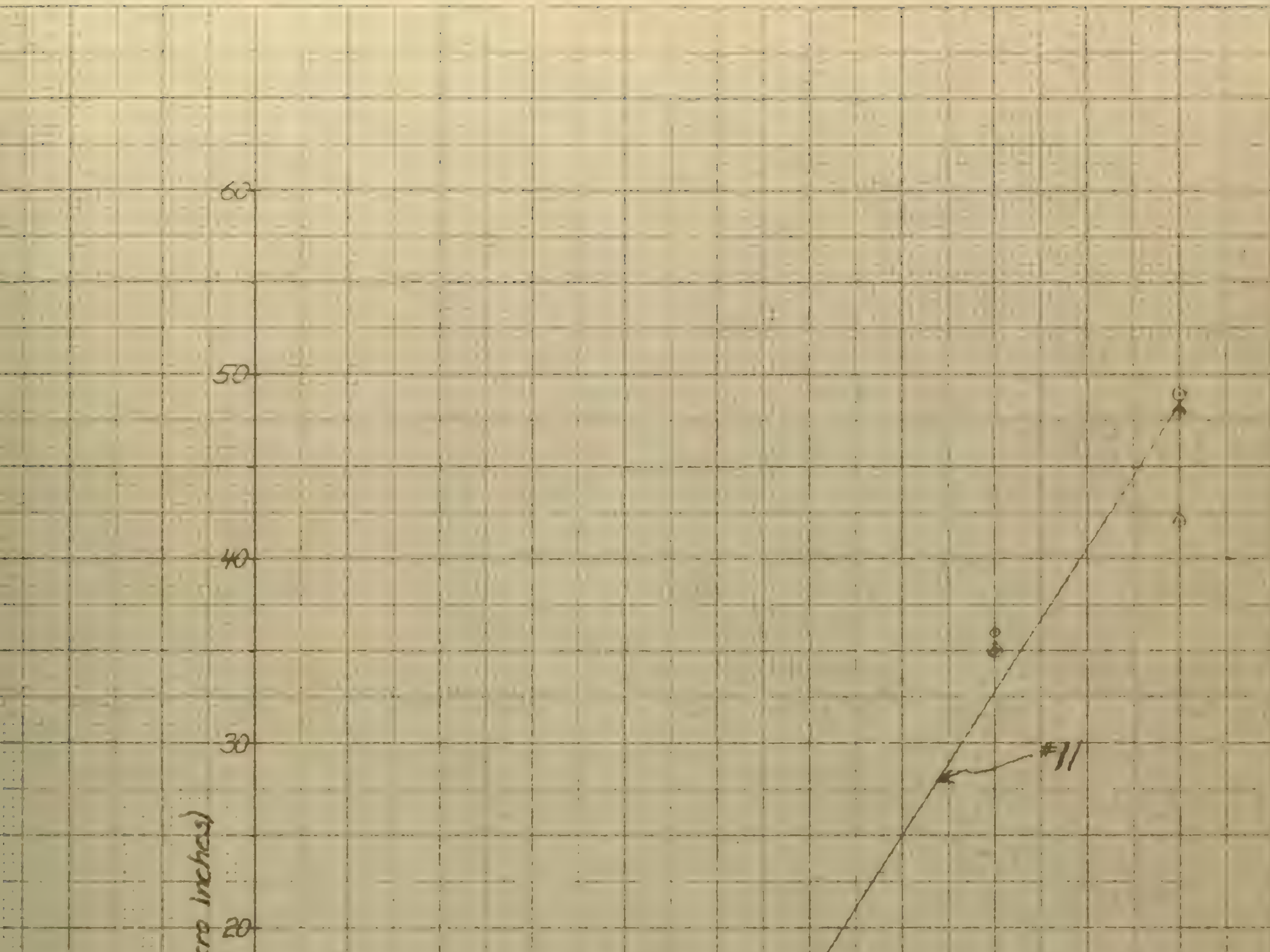


Fig. 6
Strain ~ Loading Curve
Gage # 11
(Not Corrected for G.F.)

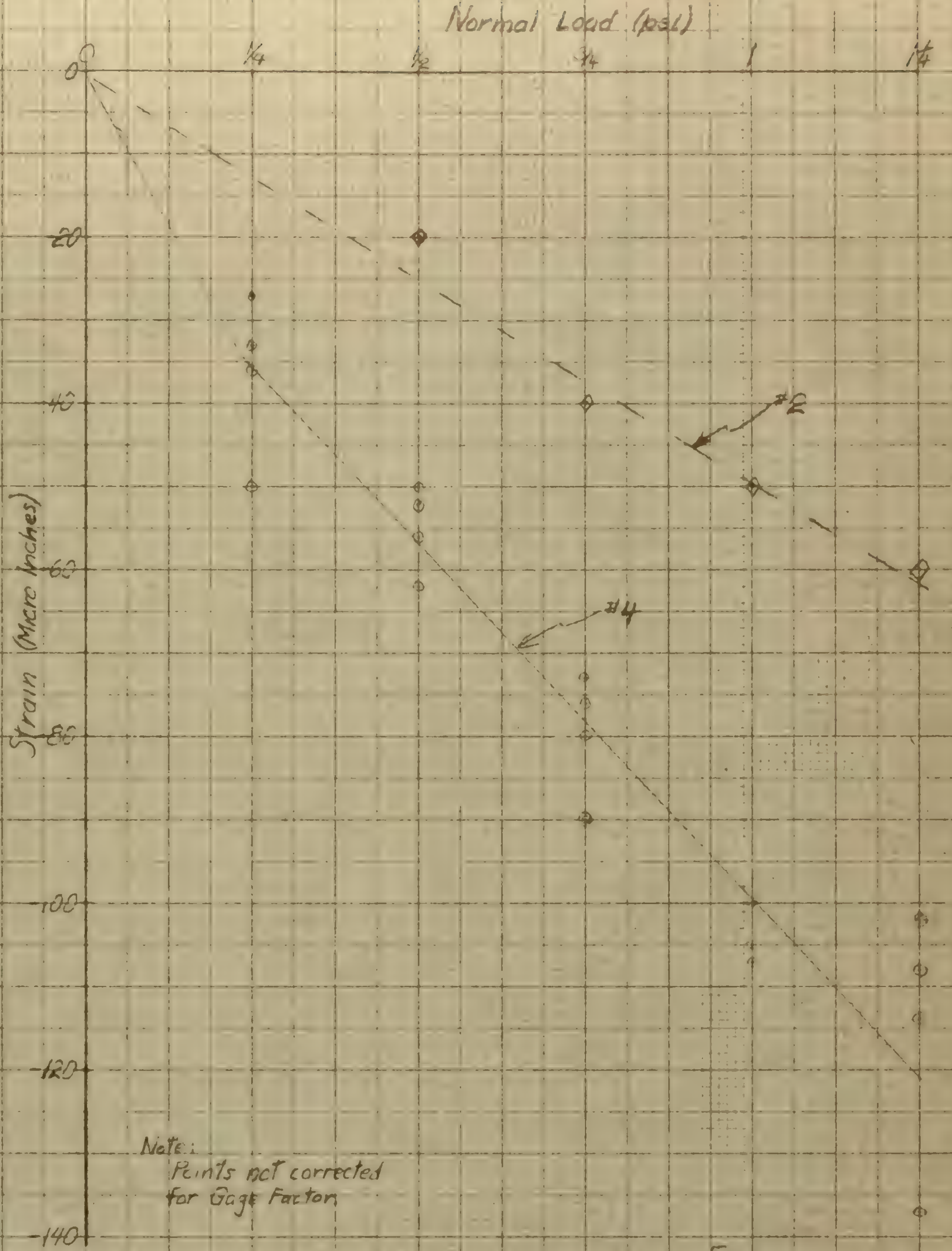


Fig. 7
Strain ~ Loading Curves
Gages #2 & #4
(at 14.125 in. radius)
CHG

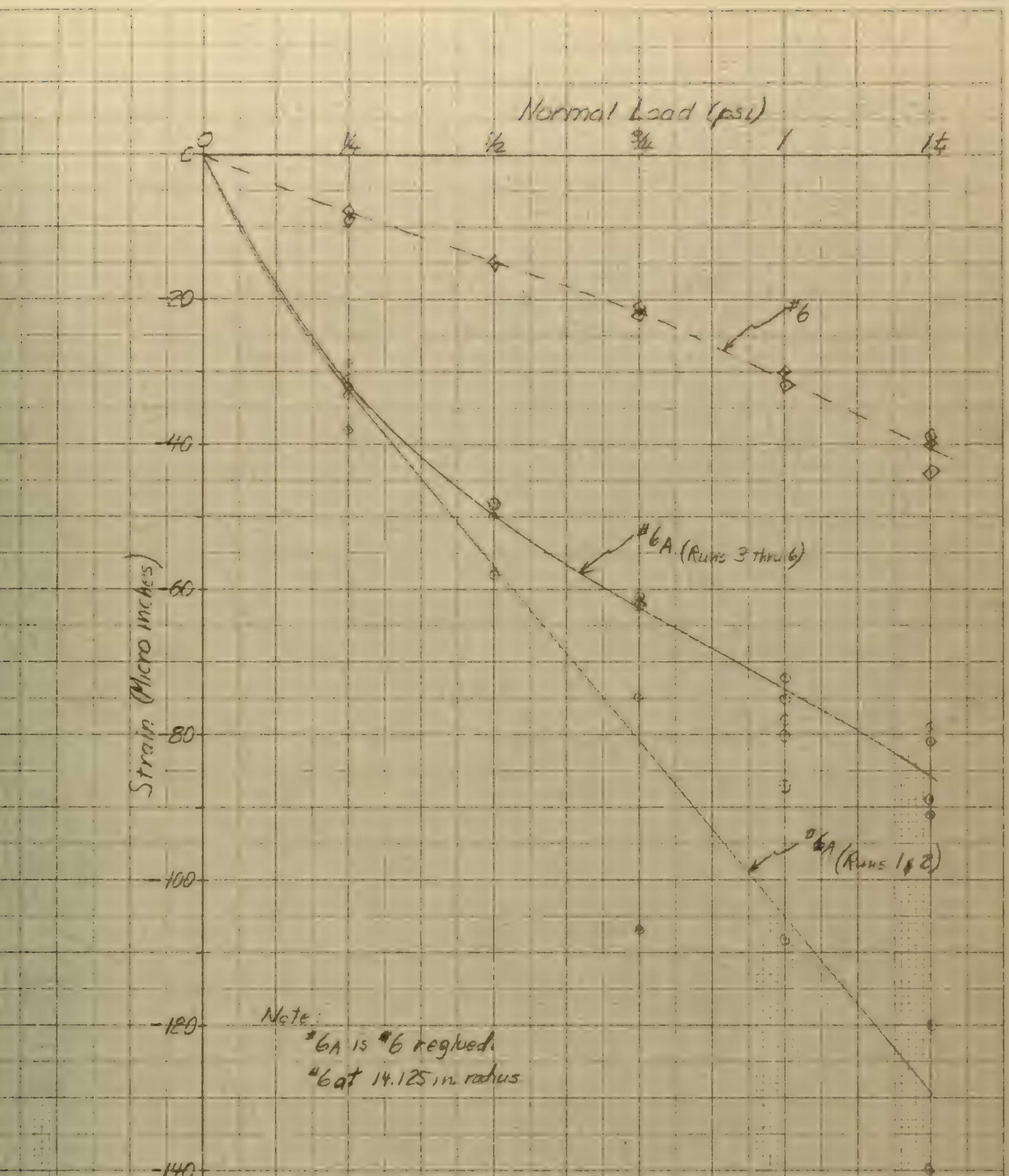


Fig. 8
Strain ~ Loading Curves
Gage #6
(Not corrected for G.F.)

CHG

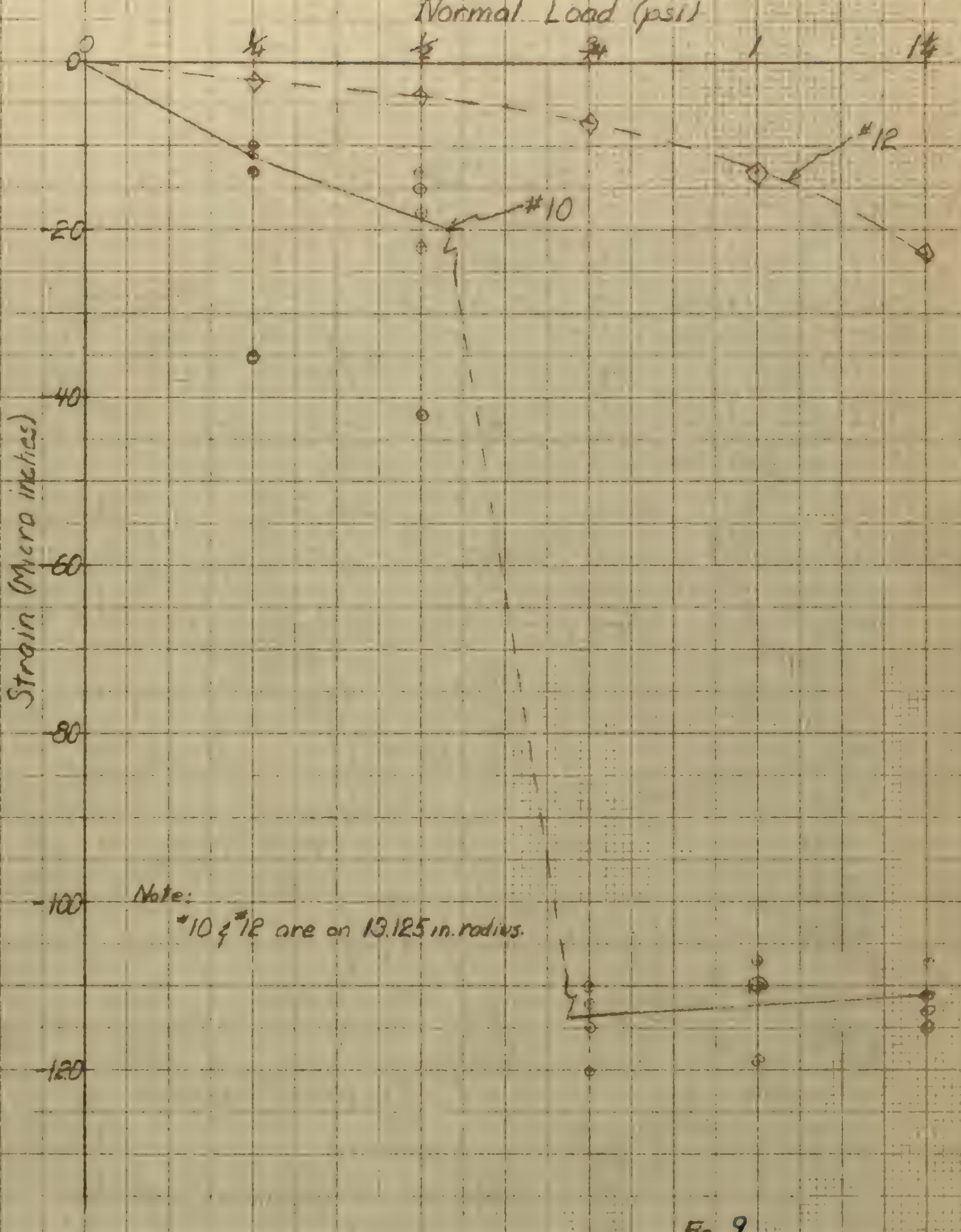


Fig. 9
Strain - Loading Curve
Gages #10 & #12
(Not corrected for G.F.)

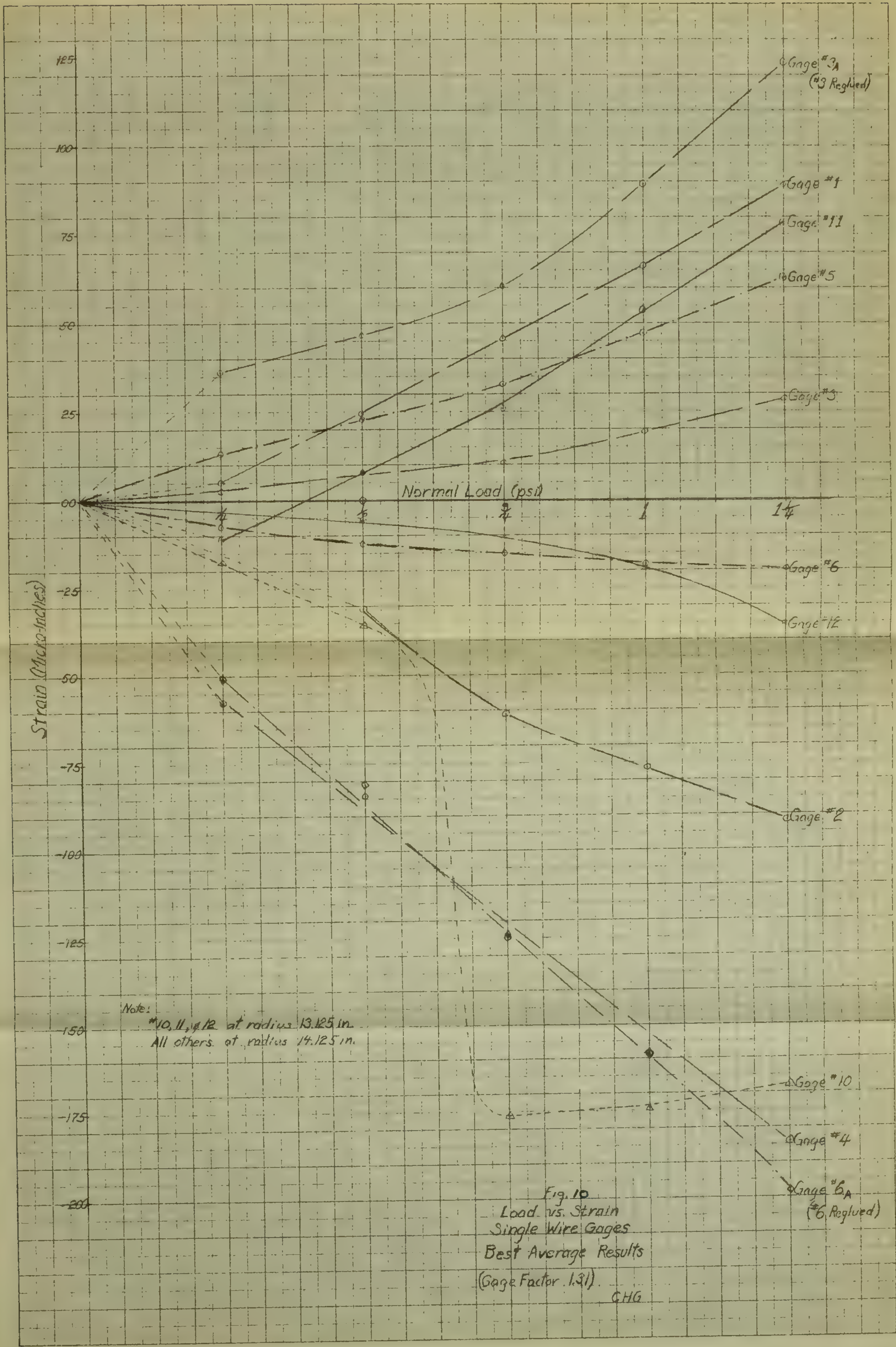


Fig. 10
Load vs. Strain
Single Wire Gages
Best Average Results
(Gage Factor 1.31)
CHG

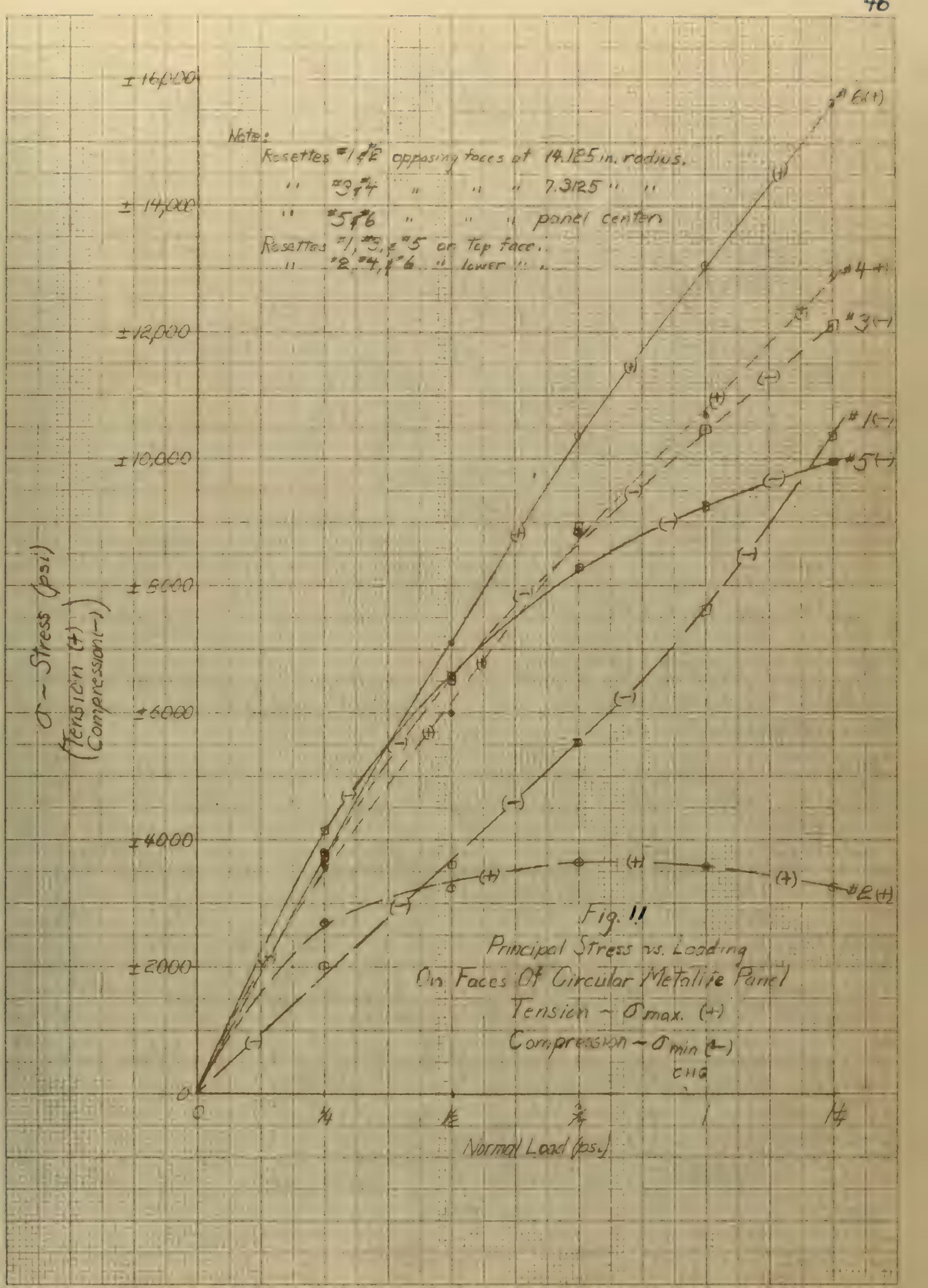
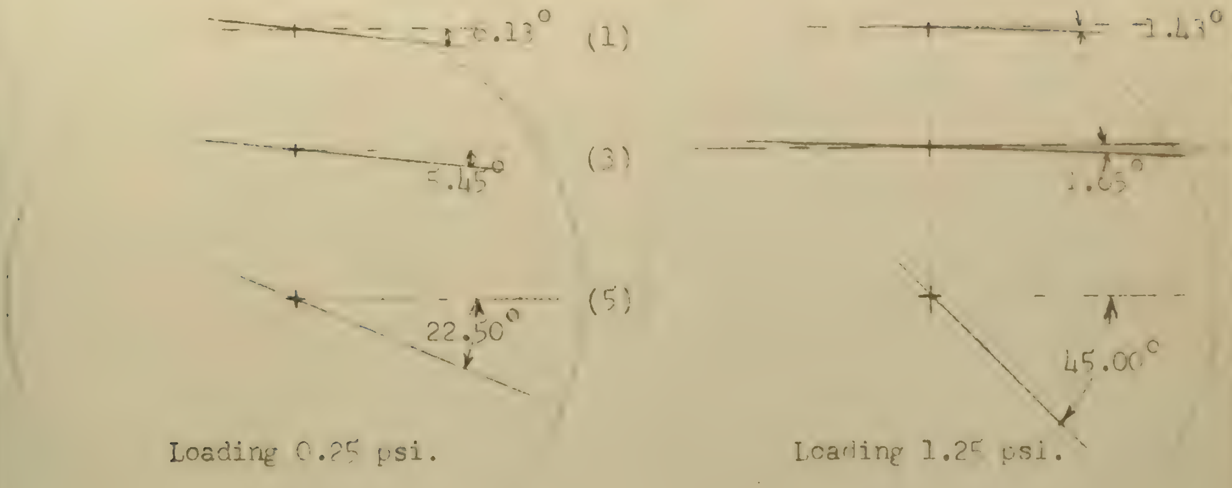


Fig. 11
Principal Stress vs. Loading
On Faces Of Circular Metalite Panel
Tension – $\sigma_{max.}$ (+)
Compression – σ_{min} (–)
CHG

Fig. 12

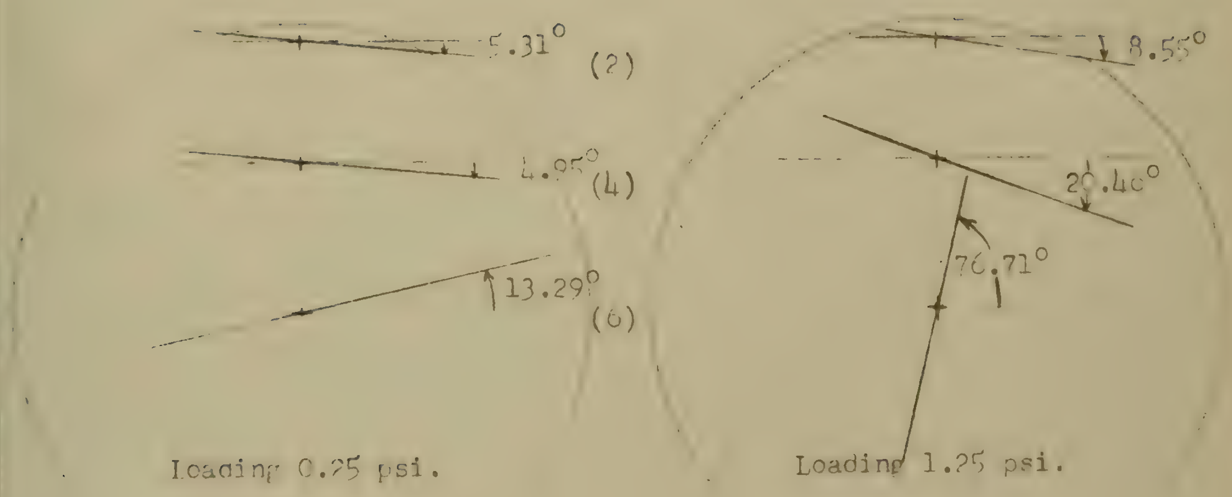
ANGLES TO PRINCIPAL AXIS

(Rosette Gages)



TOP FACE

(σ_{min} - Compression)



LOWER FACE

(σ_{max} - Tension)

( ) Rosette Numbers

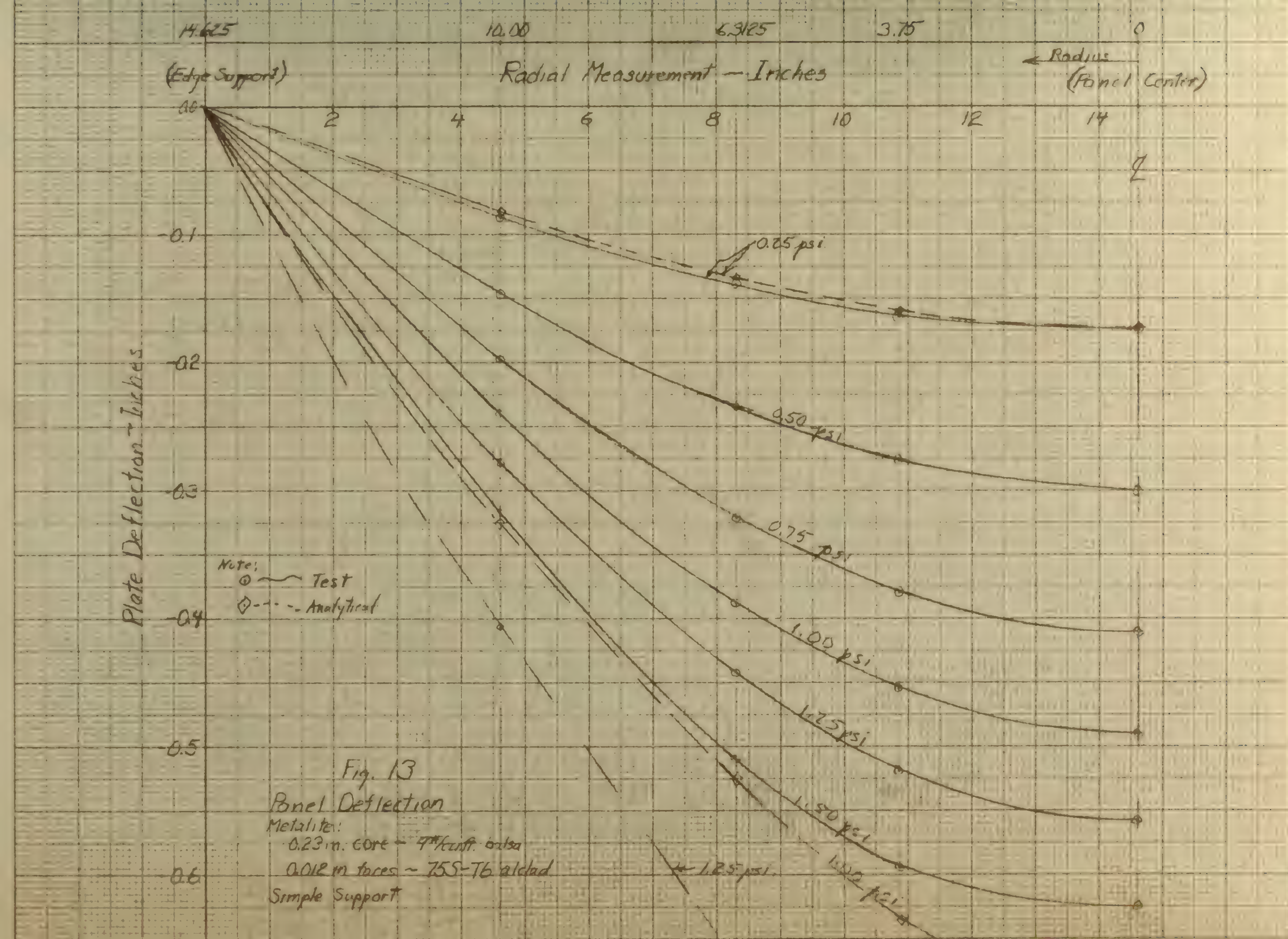


Fig. 13
Panel Deflection
Metalite:
0.23 in. core ~ 9#/cu.ft. balsa
0.012 in faces ~ 75S-T6 alclad
Simple Support

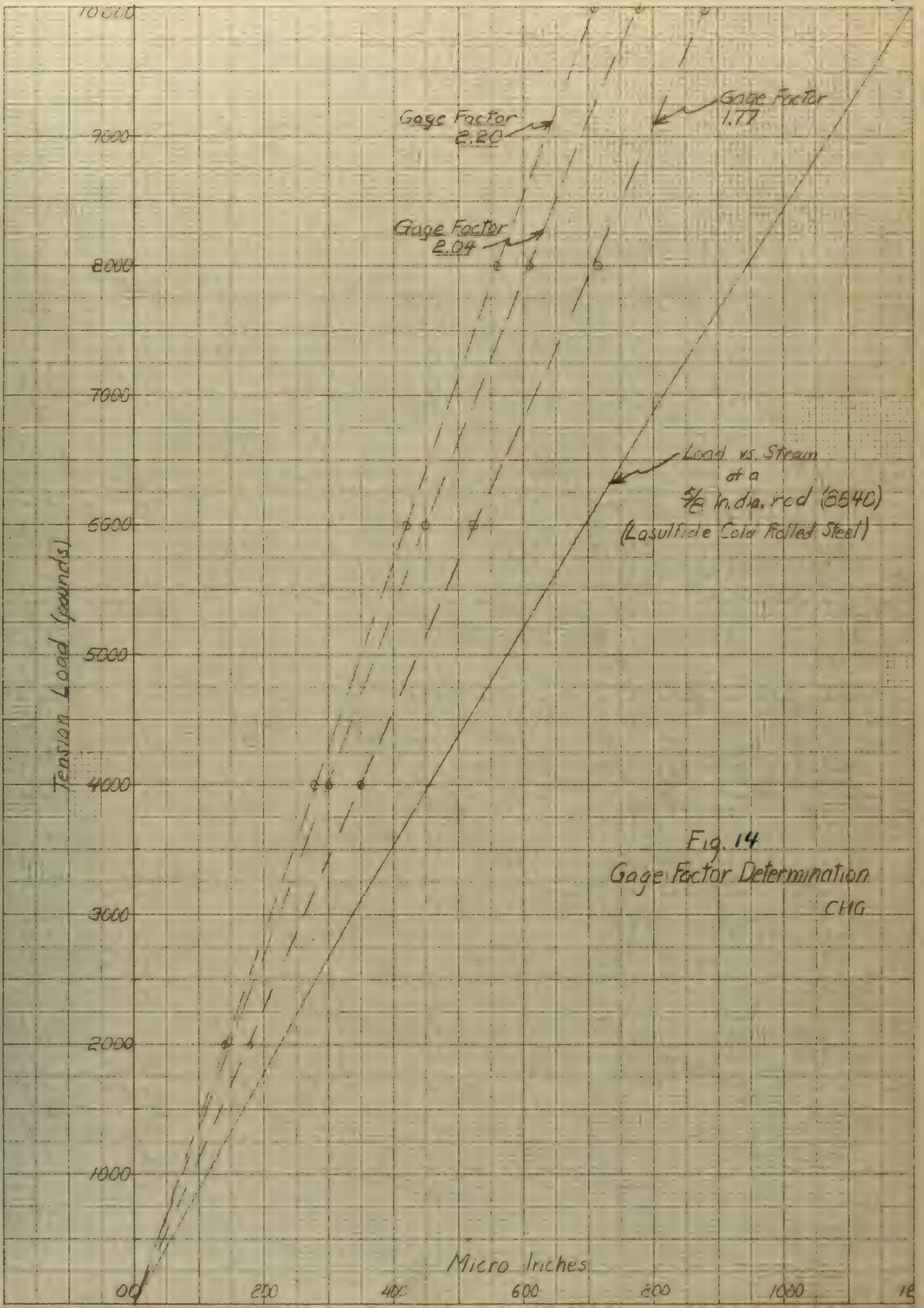


Fig. 14
Gage Factor Determination
CHG

pla